THEORY PROBAB. APPL.
Vol. 67, No. 4, pp. 652–667

ABSTRACTS OF TALKS GIVEN AT THE 7TH INTERNATIONAL CONFERENCE ON STOCHASTIC METHODS, I*

(*Translated by A. R. Alimov*)

DOI. 10.1137/S0040585X97T991210

The Seventh International Conference on Stochastic Methods (ICSM-7) was held June 2–9, 2022 at Divnomorskoe (near the town of Gelendzhik) at the Raduga sports and fitness center of the Don State Technical University. ICSM-7 was organized by the Steklov Mathematical Institute of Russian Academy of Sciences (Steklov International Mathematical Center; Department of Theory of Probability and Mathematical Statistics); Moscow State University (Department of Probability Theory); National Committee of the Bernoulli Society of Mathematical Statistics, Probability Theory, Combinatorics, and Applications; and the Don State Technical University (Department of Higher Mathematics). The conference chairman (presiding remotely) was A. N. Shiryaev, a member of the Russian Academy of Sciences, who chaired the previous three conferences and also headed the Organizing Committee and the Program Committee.

Many leading scientists from Russia, France, Portugal, and Tadjikistan took part in ICSM-7. Russian participants came from Veliky Novgorod, Voronezh, Zernograd, Kaluga, Kizil, Maikop, Moscow, Nizhni Novgorod, Rostov-on-Don, Samara, St. Petersburg, Sochi, Syktyvkar, Taganrog, Tomsk, Tyumen, Ufa, and Chelyabinsk. Approximately one-quarter of the talks were given by postgraduate and undergraduate students. Twenty-nine talks were given at joint sessions, and 36 talks were presented at parallel sessions.

Financial support from the Steklov International Mathematical Center (Steklov Mathematical Institute of Russian Academy of Sciences), Russia, was invaluable for the successful work of the conference.

At the opening of the conference, I. V. Pavlov, the Organizing Committee Deputy Chair, read the following welcome message from A. N. Shiryaev to the conference participants.

Dear Colleagues!

Despite numerous difficulties, our Rostov-on-Don probabilists have managed to gather all of us in this wonderful Black Sea city at the 7th International Conference on Stochastic Methods. This is essentially the only current large conference on probability theory, mathematical statistics, and their application.

We all know that the classical probability theory is mainly associated with the limiting theorems such as the law of large numbers, the central limit theorem, and the Poisson theorem. This topic continues to occupy a worthy place in probability theory and is presented at our conference. The limit theorems play an important role in probability theory as a link, say, between models with discrete and continuous times, and as a phenomenon that reveals the meaning of the very concept of probability.

*Published electronically February 8, 2023. Part II will be published in *Theory Probab. Appl.*, 68 (2023). The conference was supported by the Ministry of Science and Higher Education of the Russian Federation (the grant to the Steklov International Mathematical Center, agreement 075-15-2022-265). Originally published in the Russian journal *Teoriya Veroyatnostei i ee Primeneniya*, 67 (2022), pp. 819–836.
https://doi.org/10.1137/S0040585X97T991210

A large number of talks at the conference are devoted to applications of probability theory and mathematical statistics, which corresponds to the very name of the conference as a conference on stochastic methods. Last year, despite the pandemic, we successfully held the 6th conference (via Zoom) in Moscow, where representatives of all five continents participated. In total, 47 talks were given. The Moscow conference was dedicated to the bicentenary of our wonderful mathematician, Pafnutii L'vovich Chebyshev. The number of works of Chebyshev on the probability theory is small (just four). But all of them played a decisive role in the formation and maturation of probability theory.

This year, in July, the 33rd European conference on statistics, in the broad sense of the word "statistics" (including mainly its economic aspects), was supposed to be held. The World Congress of Mathematics in St. Petersburg (Russia) was also scheduled. Unfortunately, these events did not take place. We hope that, in the future, ordinary contacts with foreign colleagues will continue and we will, for example, have the opportunity to visit the 43rd conference on stochastic processes and their applications, which should be held in Lisbon (Portugal) in July 2023. In Soviet times, we actively participated in the organization of international scientific activities. In this regard, let us recall the Soviet–Japanese symposia and the first World Congress of the Bernoulli Society in Tashkent (1986). I would like to hope that following the example of our Rostov-on-Don colleagues, there will be young people who would organize conferences of young researchers. Events like summer workshops are also rare in Russia nowadays.

The next year will be marked by the 120th anniversary of the birth of A. N. Kolmogorov, and we should all work to have a successful 8th conference dedicated to this date. We would be grateful for the advice, suggestions, and help.

It remains to wish everyone successful work, and, of course, many sunny days at this beautiful Black Sea resort!

A. N. Shiryaev, I. V. Pavlov, P. A. Yaskov, T. A. Volosatova

The abstracts of the talks and presentations given at the conference are provided below.

V. I. Afanasyev (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia). **On local times of conditional random walks.** [1]

Let $X_1, X_2, \dots$ be independent r.v.'s with the same arithmetic distribution with maximal span 1, and let $\mathbf{E}X_1 = 0$, $\mathbf{E}X_1^2 := \sigma^2 \in (0, +\infty)$. Consider a random walk $S_0 = 0$, $S_i = \sum_{j=1}^{i} X_j$, $i \in \mathbf{N}$. Next, let $T = \min\{i > 0 \colon S_i \leqslant 0\}$. Consider the *stopped random walk* $\widetilde{S}_i = S_i$ for $i < T$ and $\widetilde{S}_i = 0$ for $i \geqslant T$. We set $\widetilde{\xi}(k) = |\{i \geqslant 0 \colon \widetilde{S}_i = k\}|$.

Let $\{W^+(t),\, t \geqslant 0\}$ be a Brownian meander and $l^+(u)$ be its local time, i.e., $l^+(u) = \lim_{\varepsilon\to 0} \varepsilon^{-1} \int_0^{+\infty} I_{[u,u+\varepsilon]}(W^+(s))\, ds$ for $u > 0$.

THEOREM 1. *As $n \to \infty$,*

$$\left\{ \frac{\sigma\widetilde{\xi}(\lfloor u\sigma\sqrt{n}\rfloor)}{\sqrt{n}},\, u \geqslant 0 \,\middle|\, T > n \right\} \to \{l^+(u),\, u \geqslant 0\},$$

where the symbol $\to$ means the convergence in distribution in the space $D[0, +\infty)$ with the Skorokhod topology.

[1] This work was performed at the Steklov International Mathematical Center with the support of the Ministry of Science and Higher Education of the Russian Federation (agreement 075-15-2019-1614).

Let $\{W_0^{\uparrow}(t),\, t \geqslant 0\}$ be a Brownian high jump and $l_0^{\uparrow}(u)$ be its local time, i.e., $l_0^{\uparrow}(u) = \lim_{\varepsilon\to 0} \varepsilon^{-1} \int_0^{+\infty} I_{[u,u+\varepsilon]}(W_0^{\uparrow}(s))\, ds$ for $u > 0$. We set $T_x = \min\{i\in\mathbf{N}\colon \widetilde{S}_i > x\}$ for $x > 0$.

THEOREM 2. *As* $n \to \infty$,

$$\left\{\frac{\sigma^2\widetilde{\xi}(\lfloor un\rfloor)}{n},\, u \geqslant 0 \,\middle|\, T_n < +\infty\right\} \to \{l_0^{\uparrow}(u),\, u \geqslant 0\}.$$

L. G. Afanasyeva (The author is deceased. Former address: Lomonosov Moscow State University, Moscow, Russia)**, E. E. Bashtova** (Lomonosov Moscow State University, Moscow, Russia)**. Asymptotic analysis of systems with orbit with regenerative input flow.** [2]

Consider an m-channel queuing system. The service times $\{\eta_i\}_{i=1}^{\infty}$ on each server are i.i.d. r.v.'s. If there is at least one free server at the moment of customer arrival, then the customer is processed immediately. If all the servers are busy, then the customer is sent to the so-called orbit, from where it repeats attempts to get to service. We study the process $Q(t)$, which is the number of customers in the system. It is assumed that the input flow $X(\,\cdot\,)$ is a regenerative flow with regeneration periods $\{\tau_i\}_{i=1}^{\infty}$ and increments over the regeneration period $\{\xi_i\}_{i=1}^{\infty}$. Units from the orbit with j customers arrive in a Poisson stream of intensity $\nu(j)$. Conditions for ergodicity of such systems were obtained in [1]. We prove the following theorem for overloaded systems.

THEOREM. *Let* $\rho_I = \lambda b/m > 1$, $\mathbf{E}\tau_i^r < \infty$, $\mathbf{E}\xi_i^r < \infty$, $\mathbf{E}\eta_i^r < \infty$ *for some* $r > 2$ *and* $j^{-1+1/r}\nu(j) \to \infty$ *as* $j \to \infty$. *Then there exists a standard Wiener process* W *such that* $\sup_{0\leqslant u\leqslant t} \|Q(u)-(\rho_I-1)u-\sigma_I W(u)\| = o(t^{1/r})$ *a.s., where* $\sigma_I^2 = \sigma_X^2 + m\sigma_S^2$.

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E. V. Alymova (Russian Customs Academy, Rostov branch, Rostov-on-Don, Russia)**. Software implementation and statistical estimate for equivalence of models of classification of symbols in Latin alphabet based on impulse and convolution neural networks.**

We consider the problem of recognition of 62 printed characters of the Latin alphabet by convolution [1] and impulse [2] neural networks. 62 datasets of 983 images of 128×128 points are formed for 26 letters (in both registers) and 10 digits.

The convolution network is based on Tensorflow and contains nine layers and 97,278 training parameters. On average, 89 characters out of 100 are recognized correctly, i.e., the recognition accuracy reaches 89%. We single out sets of symbols within which a symbol is assumed to lie in a single class: $\{\mathrm{i, l, j, 1}\}$, $\{\mathrm{g, 9}\}$, $\{\mathrm{c, C}\}$, $\{\mathrm{p, P}\}$, $\{\mathrm{o, 0, O}\}$. Subject to this proviso, the convolution network recognizes, on average, 93 symbols out of 100, i.e., its mean accuracy is 93%.

[2] Supported by the Russian Foundation for Basic Research (grant 20-01-00487).

The impulse neural network is based on NengoDL. With the above union of classes, the impulse neural network recognizes, on average, 91 symbols out of 100, i.e., its mean accuracy is 91%.

THEOREM. *The performance of the model of classification of symbols of the Latin alphabet with an impulse neural network with* $\chi^2 = 2.4671$, $P_{\text{value}} = 0.07437$ *for the McNemar test, and* 5% *significance level is similar to that based on a convolution network.*

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A. M. Atayan (Don State Technical University, Rostov-on-Don, Russia)**. Evaluation of the operating time of a computing system based on correlation analysis.** [3]

To increase the accuracy of mathematical models related to solution of problems in hydrophysics, such models should include factors with considerable effect on the underlying processes [1]. Calculation time can be considerably reduced with the use of multiprocessor systems. However, the time efficiency of a computing system operating time may be far from expected. This calls for a theoretical analysis of the calculation time based on correlation analysis.

THEOREM. *A multiple regression model is considered. Let* t_i *be the total execution time of a computation system* (*in seconds*), *and let* n_i, k_i *be the explanatory factors* (*the volume of the transmitted data and the number of utilized computational nodes, respectively*)*. Then the latency time can be found by*

$$t_i = \begin{cases} 5.21 \cdot 10^{-6} + 1.53 \cdot 10^{-7} k_i, & n_i \leqslant 512, \\ 6.733 \cdot 10^{-6} k_i, & n_i > 512, \end{cases} \qquad \textit{where} \quad i = 1, 2, \dots, p.$$

The transfer time of a single datum is $t_x = 3.3 \cdot 10^{-9}$ s.

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E. E. Bashtova (Lomonosov Moscow State University, Moscow, Russia)**. On strong approximation of some types of random flights.** [4]

Let $\varepsilon = \{\varepsilon_n,\, n \geqslant 0\}$ be i.i.d. random vectors on the unit sphere in $\mathbf{R}^k$, and let $\{T_n,\, n \geqslant 0\}$ be an increasing sequence of r.v.'s independent of ε $(T_0 = 0)$. A random flight (see [3]) is a continuous random process $X = \{X(t),\, t \geqslant 0\}$, whose trajectory

[3] Supported by the Russian Foundation for Basic Research (grant 20-31-90105).
[4] Supported by the Russian Foundation for Basic Research (grant 20-01-00487).

on the interval $[T_{n-1}, T_n]$ $(n \geqslant 1)$ is linear and whose direction is given by realization of the random vector ε_n, $\mathbf{E}\varepsilon_1 = 0$, $\mathbf{D}\varepsilon_1 = \sigma_\varepsilon^2$. Let $N(t) = \min\{n \geqslant 0\colon T_n > t\}$, $t \geqslant 0$.

THEOREM. *Let $N(t)$ be a regenerative flow* [1], [2]. *If $\mathbf{E}e^{p\tau_i} < \infty$ for some $p > 0$, then, on the same probability space, one can define a process X and a d-dimensional Wiener process $\{W_t,\, t \geqslant 0\}$ so that*

$$\mathbf{P}\Bigl(\sup_{u\leqslant t}|X(u) - \sigma_\varepsilon\sqrt{\mathbf{E}\tau_1}\,W(u)| > a\ln t + x\Bigr) \leqslant be^{-cx}$$

for all $t \geqslant 1$, $x > 0$ and some constants $a, b, c > 0$.

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A. F. Beknazaryan (University of Tyumen, Tyumen, Russia)**, H. Sang** (The University of Mississippi, Oxford, MS, USA)**, Y. Xiao** (Michigan State University, East Lansing, MI, USA)**. Cramér type moderate deviations for random fields.**

Let $\{X_{nj},\ n \in \mathbf{N},\, j \in \mathbf{Z}^d\}$ be a random field with zero means, and let, for each n, the cumulant generating functions $L_{nj}(z) = \ln \mathbf{E}e^{zX_{nj}}$ of the r.v.'s X_{nj}, $j \in \mathbf{Z}^d$, be analytic in the disk $|z| < H_n$ in which $|L_{nj}(z)| \leqslant c_{nj}$. Assume that X_{nj}, $j \in \mathbf{Z}^d$, are independent for each $n \in \mathbf{N}$ and that $S_n = \sum_{j\in\mathbf{Z}^d} X_{nj}$, $B_n = \sum_{j\in\mathbf{Z}^d} \mathbf{D}X_{nj}$, $C_n = \sum_{j\in\mathbf{Z}^d} c_{nj}$, and $F_n(x) = \mathbf{P}(S_n < x\sqrt{B_n}\,)$ are well defined and finite, where $B_nH_n^2 \to \infty$ and $C_n = O(B_nH_n^2)$. Let $\Phi(x)$ be the distribution function of the standard normal r.v. We prove the following theorem that extends [1] and [2].

THEOREM. *Let $x \geqslant 0$ and $x = o(H_n\sqrt{B_n}\,)$. Then*

$$\frac{1-F_n(x)}{1-\Phi(x)} = \exp\biggl\{\frac{x^3}{H_n\sqrt{B_n}}\,\lambda_n\biggl(\frac{x}{H_n\sqrt{B_n}}\biggr)\biggr\}\biggl(1 + O\biggl(\frac{x+1}{H_n\sqrt{B_n}}\biggr)\biggr),$$

where $\lambda_n(t)$ is a power series that converges for sufficiently small $|t|$ uniformly with respect to n.

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G. I. Beliavsky, N. V. Danilova, G. A. Ougolnitsky (Southern Federal University, Rostov-on-Don, Russia). **A game model of investment portfolio management.**

The optimal portfolio problem is considered as an N-round stochastic Stackelberg game between two players, one of whom (unit investment fund (UIF)) is engaged in investments, and the other (agent) allocates funds for these investments. The leading player (UIF) forms a portfolio Z and stimulates receipts of funds from the agent, providing the agent with profitability K exceeding the known value g with probability $P(Z)$. An optimal strategy of the agent for probability $P(Z)$ of portfolio Z is a binary mixed strategy $x(Z)$ with probability distribution $\mathbf{P}(x(Z)=1)=P(Z)$ and $\mathbf{P}(x(Z)=0)=1-P(Z)$. Thus, the Stackelberg equilibrium in this game is reached on $\langle Z^*, x(Z^*)\rangle$. Here, Z^* is the solution of the optimization problem $\max_{(Z,I)=1} P(Z)$. In order to achieve the Stackelberg equilibrium, both the UIF and the agent rely on real-time learning by solving the following stochastic programming problems. The problem of the UIF is to calculate the value $\min_{(Z,I)=1} \mathbf{E}_R(g+1-\alpha(Z,R))^+$, where $\alpha \in (0,1)$ is the asset return vector, while the UIF monitors the sequence of return vectors. The agent calculates $\min \mathbf{E}_\delta(\delta - y)^2$ by observing the sequence $\delta_i = I_{\{K_i \geqslant g\}}$. The following result is used: $\mathbf{E}_\xi(g+1-\xi)^+ \geqslant \mathbf{P}(\xi \leqslant g)$.

Ya. I. Belopolskaya (Sirius University, Sochi, Russia; St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, Russia). **Probabilistic representation of a solution to Cauchy–Robin problem for a system of nonlinear parabolic equations.** [5]

We prove the following result, which extends our earlier result for $d_1 = 1$.

THEOREM. *Let $A_q \in \mathbf{R}^d \otimes \mathbf{R}^d$, $c_q \in \mathbf{R}$ be smooth bounded functions, $A_q(t,x,u) = \int_{\mathbf{R}^d_+} \sum_{m=1}^{d_1} A_{qm}(x-y) u_m(t,y)\,dy$, ρ be a mollifier, and ξ_{0q} be independent r.v.'s, which do not depend on independent Wiener processes $w_q(t) \in \mathbf{R}^d$. Then there exists a unique solution $(\xi_q(t), k_q(t), u_q(t,y))$ of the system*

$$\xi_q(t) + k_q(t) = \xi_{0q} + \int_0^t A_q(s, \xi_q(s), u)\,dw_q(s), \qquad \mathbf{P}\{\xi_{0q} \in dy\} = u_q(0,y)\,dy,$$

$$k_q(t) = \int_0^t n(\xi_q(s))\,d|k_q|(s), \qquad |k_q|(t) = \int_0^t I_{\partial G}(\xi_q(s))\,d|k_q|(s),$$

$$u_q(t,y) = \mathbf{E}\biggl[\rho(y - \xi_q(t)) \exp\biggl\{\int_0^t c_q(s, \xi_q(s), u)\,ds\biggr\}\biggr], \qquad q = 1, \dots, d_1,$$

and $v_q(t,y) = \int_G \rho(y-x) u_q(t,x)\,dx$ satisfy weakly the problem

$$\frac{\partial v_q}{\partial t} = \frac{1}{2} \operatorname{Tr} \nabla^2[(A_q A_q^*)(y,v) v_q(t)] + c_q(y,v) v_q(t,y), \qquad y \in G = \mathbf{R}^d_+,$$

$$v_q(0,y) = v_{0q}(y), \quad y \in G, \qquad \sum_{k=1}^d \nabla_{y_k}[(A_q A_q^*)(y,v) v_q] n_k(y) = 0, \quad y \in \partial G.$$

[5]Supported by the Russian Science Foundation (grant 22-21-00016).

D. V. Bondarenko, A. V. Nikitina (Don State Technical University, Rostov-on-Don, Russia). **Evaluation of the effectiveness of heuristic optimization methods with random distribution of input data.** [6]

We prove the following theorem, which involves the pollutant concentration function from [1].

THEOREM. *Let the pollutant concentration function have the form*

$$C(x,y) = \begin{cases} \sin\dfrac{\pi(x-10)}{10}\sin\dfrac{\pi(y-10)}{10}, & (x,y)\in D,\\ 0, & (x,y)\notin D, \end{cases}$$
$$D = \{(x,y)\in \mathbf{R}^2\colon x\in[10,20],\ y\in[10,20]\}.$$

Then the heuristic method of harmonic search with random distribution of input data produces a global maximum point $(15.00057081044254, 14.9991222560702)$ *where the function is equal to* 0.99999994590178. *The execution time is* 0.496674 *seconds under the condition that the limit on the number of improvisations in the iterative part is set to* 10,000, *and the number of harmonics that can be stored in memory is set to* 100. *The probability of choosing from memory harmonics* p_c *is* 0.8, *and the probability of modification* p_m *is* 0.1.

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A. E. Chistyakov (Don State Technical University, Rostov-on-Don, Russia), **I. Yu. Kuznetsova** (Southern Federal University, Rostov-on-Don, Russia). **Stability estimation of the equation for pressure calculation with due account of the collision time of medium molecules.** [7]

We propose an approach to constructing a mathematical model of hydrodynamics based on a relation between the kinetic and hydrodynamic descriptions of continuous fluid. According to [1], in the case of a spatially one-dimensional layer, there is a constant locally Maxwellian distribution $f_{0i} = \rho_i \exp\{-(\zeta - U_i)/(2RP_i)\}/(2RP_i)^{3/2}$, where ρ_i is the density of substance, R is the gas constant, P_i is the pressure, ζ is the molecule velocity, and U_i is the macroscopic velocity, at the nth time step in ith spatial cell. It is known (see [1]) that, for solution of hydrodynamics problems, the continuity equation can be augmented with the term $\tau^*\rho''_{tt}$ appearing if the delay in the transmission of momentum is taken into account when the collision of molecules can be represented by a discrete function, where $\tau^* \sim h/c$ is the regularization parameter or the characteristic time between collisions of molecules, h is the computational grid step, and c is the speed of sound.

THEOREM. *The implicit difference scheme approximating the homogeneous pressure equation* $P''_{tt}/c^2 - \Delta P = -(\rho'_t + \nabla(\rho\widetilde{\mathbf{V}}))/\tau$, *where* τ *is the time step, and* $\widetilde{\mathbf{V}}$ *is the intermediate velocity field calculated without regard to pressure, is absolutely stable and has the first order of accuracy.*

[6]The study was carried out with partial financial support from the Council for Grants of the President of the Russian Federation (project MD-3624.2021.1.1).

[7]The study was carried out with partial financial support from the Council for Grants of the President of the Russian Federation (project MD-3624.2021.1.1).

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E. G. Chub, V. A. Pogorelov (Don State Technical University, Rostov-on-Don, Russia)**. Stochastic nonlinear dynamics model of an information-measuring complex gyrostabilizer of a track-testing car.**

We study the dynamics of motion of a gyrostabilized information-measuring complex of a track-testing car (IMCTTC) under the action of interference [1], [2], [3]. A moment method is applied for constructing a moment approximation of the posterior probability density of IMCTTC for the stochastic model of a gyrostabilized IMCTTC in the "object–observer" form $\dot{Y} = F(Y,t) + F_0(Y,t)\xi$, where Y is the state vector describing the object dynamics, F, F_0 are known nonlinear functions determined from IMCTTC performance conditions, and ξ is the vector of random disturbing accelerations described, in general, by Gaussian noise with zero expectation and a known intensity matrix. Our method is capable of enhancing the performance of advanced IMCTTCs.

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A. G. Danekyants, N. V. Neumerzhitskaia, I. V. Pavlov, I. V. Tsvetkova (Don State Technical University, Rostov-on-Don, Russia)**. Some results on signed interpolating deflators.**

We continue the study of signed interpolating deflators started in [1]. Consider a stochastic basis $\bigl(\Omega, F = (\mathcal{F}_k)_{k=0}^K, \mathbf{P}\bigr)$, where Ω is a finite set, $K < \infty$, and $\mathcal{F}_0$ is trivial. Let A be an atom in $\mathcal{F}_k$ $(0 \leqslant k < K)$, let B_i $(i = 1, 2, \dots, m)$ be atoms in $\mathcal{F}_{k+1}$, and let

$$A = B_1 + B_2 + \dots + B_m, \quad a := Z_k|_A, \quad b_i := Z_{k+1}|_{B_i}, \quad p_i := \mathbf{P}(B_i), \quad d_i := D_{k+1}|_{B_i}$$

(in general, the splitting index m of an atom A and the numbers a, b_i, p_i, d_i depend on A). A signed deflator $D = (D_k, \mathcal{F}_k, \mathbf{P})_{k=0}^K$ of a process $Z = (Z_k, \mathcal{F}_k)_{k=0}^K$ is called admissible if $\sum_{i \in I} p_i d_i \neq 0$ for any $0 \leqslant k < K$, for any atom $A \in \mathcal{F}_k$, and for any nonempty set $I \subset \{1, 2, \dots, m\}$.

Theorem. *Let $m \geqslant 2$ for any k, $0 \leqslant k < K$, and any atom $A \in \mathcal{F}_k$, and let $a, b_1, \dots, b_m$ be distinct. Then there exists an admissible signed deflator D satisfying the universal Haar uniqueness property* (*UHUP*) (*see* [1])*. If D is positive, then the UHUP for D coincides with the UHUP for the martingale measure of the process Z corresponding to the deflator D.*

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D. V. Dimitrov (Lomonosov Moscow State University, Moscow, Russia)**. Statistical methods for determining heterogeneities of fibrous materials from statistics of nearest neighbors.** [8]

Estimates of nearest neighbors with Kullback–Leibler divergence [1] are applied to searching heterogeneity domains in fibrous materials. A model is studied with an observer knowing independent r.v.'s $\{X_i, T_i,\, i \in \{1, \dots, \zeta_n\}\}$, where X_i with values in $\mathbf{R}^{d_1} \cap \Pi$ is the center of the ith fiber, $\Pi \in \mathcal{B}(\mathbf{R}^{d_1})$ is the ambient bounded material, $\operatorname{law}(X_i) = \operatorname{law}(X)$; $T_i = \xi_i \cdot \mathbf{I}\{X_i \in R_0\} + \eta_i \cdot \mathbf{I}\{X_i \in \Pi \setminus R_0\}$ with values in $\mathbf{R}^{d_2}$ is the label (direction) of the ith fiber, $\operatorname{law}(\xi_i) = \operatorname{law}(\xi)$, $\operatorname{law}(\eta_i) = \operatorname{law}(\eta)$, $R_0 \in \mathcal{B}(\mathbf{R}^{d_1}) \cap \Pi$ is the true heterogeneity domain, and $\zeta_n \sim \operatorname{Pois}(\lambda_n \cdot \mu(\Pi))$, $\lambda_n > 0$, $\lambda_n \to \infty$, $n \to \infty$. For each set R from some family of sets $\mathcal{R}$, we construct a statistic $\widehat{T}_n(R)$ based on estimation of nearest neighbors with Kullback–Leibler divergence between the distribution of labels T_i of fibers inside a screen R (i.e., for $X_i \in R$) and outside it. Next, it is assumed that $\widehat{R}_n := \operatorname{argmax}_{R \in \mathcal{R}} \widehat{T}_n(R)$.

Theorem. *Assume that, for some $\varepsilon, R > 0$, $N \in \mathbf{N}$, $f \in \{p_\xi, p_\eta\}$, $g \in \{p_\xi, p_\eta\}$, the functionals $K_{f,g}(2, N)$, $Q_{f,g}(\varepsilon, R)$, $T_{f,g}(\varepsilon, R)$ are finite. Then*

$$\mathbf{P}\Big(\widehat{R}_n \in \operatorname*{argmax}_{R \in \mathcal{R}} d_\Pi(R, R_0)\Big) \to 1, \qquad n \to \infty.$$

Here, $d_\Pi(R, R_0) := |\mu(RR_0)/\mu(R) - \mu(\overline{R}R_0)/\mu(\overline{R})|$, $\overline{R} := \Pi \setminus R$, and p_ξ and p_η are densities of the r.v.'s ξ and η, respectively; the definitions of the functionals K_{f_1,f_2}, Q_{f_1,f_2}, T_{f_1,f_2} for densities f_1, f_2 can be found in [1].

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M. L. Esquível (FCT Nova and CMA, New University of Lisbon, Portugal)**, N. P. Krasii** (DSTU, Rostov-on-Don, Russia)**. On structured random matrices defined by matrix substitutions.**

We introduce matrix substitutions (see [1]) as a way to define structured nonrandom matrices of arbitrarily large size, which can be considered as generating random matrices having a structure and independent entries. We have a theorem of convergence in law for random matrices (see [2]).

Consider $\mathcal{M}_{+\infty} := \{M = [a_{ij}]_{i,j \geqslant 1} \colon a_{ij} \in \mathbf{Z}_p\} = \mathbf{Z}_p^{(\mathbf{N}\setminus\{0\} \times \mathbf{N}\setminus\{0\})}$.

Theorem (convergence in law of random structured matrices). *Assume that*

(a) $\sigma \colon \mathbf{Z}_p \to \mathcal{M}^{<\infty}_{d \times d}(\mathbf{Z}_p)$ *is a global substitution map*;

(b) Φ_σ *is the corresponding matrix substitution map defined on* $\mathcal{M}_{+\infty}$;

[8] This work was supported by the grant "Modern Problems of Fundamental Mathematics and Mechanics" at Lomonosov Moscow State University.

(c) *M_∞ is a fixed point of Φ_σ such that $M_0 \in \mathcal{M}_{+\infty}$ and $M_n = \Phi_\sigma(M_{n-1})$, $n \geqslant 1$, $M_\infty = \lim_{n\to+\infty} M_n$.*

Let $M_n(X_\#)$ and $M_\infty(X_\#)$ be random structured matrices with M_n and M_∞ skeletons, respectively. Then

$$\mathrm{Law}(M_n(X_\#)) \xrightarrow[n\to+\infty]{} \mathrm{Law}(M_\infty(X_\#)).$$

As an application we define a random surface canonically associated with a random matrix fixed point having a skeleton fixed point of a matrix substitution map.

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A. M. Fedotkin, N. S. Markina (National Research Lobachevsky State University of Nizhnii Novgorod, Russia). **Cyclic algorithm with extension and afterservice for management of conflict flows of heterogeneous customers.**

A mathematical model of a real system of cyclic management of conflict flows of heterogeneous customers was studied in [1]. A mathematical model of flows of this kind was constructed and studied in [2]. For $j = 1, 2$, a server in states $\Gamma^{(2j-1)}$ or $\Gamma^{(2j)}$ processes and respectively afterservices only the flow Π_j. A change of the current state of the server or its extension occurs at random moments τ_i, $i = 0, 1, \dots$. The random sequence $\{(\Gamma_i, \varkappa_{1,i}, \varkappa_{2,i}, \xi'_{1,i-1}, \xi'_{2,i-1}); i = 0, 1, \dots\}$, where Γ_i is the server state in the interval $[\tau_i, \tau_{i+1})$, $\varkappa_{j,i} \geqslant 0$ is the flow queue size, Π_j at time τ_i, and $\xi'_{j,i} \geqslant 0$ is the number processed customers in the flow Π_j on the interval $[\tau_i, \tau_{i+1})$, is a mathematical model of such systems.

Theorem. *The sequence $\{(\Gamma_i, \varkappa_{1,i}, \varkappa_{2,i}, \xi'_{1,i-1}, \xi'_{2,i-1}); i = 0, 1, \dots\}$ with given initial distribution of the vector $(\Gamma_0, \varkappa_{1,0}, \varkappa_{2,0}, \xi'_{1,-1}, \xi'_{2,-1})$ is a homogeneous multivariate Markov chain.*

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Yu. E. Gliklikh (VSU, Voronezh, Russia). **On a certain stochastic equation with mean derivatives, connected with hydrodynamics.** A description of the backward mean derivative D_*, the symmetric mean derivative (the current velocity) D_{S}, and the group of Sobolev H^s-diffeomorphisms of a flat n-dimensional torus, $s > n/2 + 2$, can be found in [1].

On the group of H^s-diffeomorphisms of a flat n-dimensional torus, we construct a process $W^{(\sigma)}(t)$ from a standard Wiener process $\sigma w(t)$ in $\mathbf{R}^n$, where $\sigma > 0$ is a constant. By F we denote the regression on the torus constructed from $D_* D_*(\sigma w(t))$.

This vector field is a vector in the tangent space in the unit e to the group of H^s-diffeomorphisms. By right shifts, we translate this vector onto the entire group, thereby obtaining the right-invariant vector field $\overline{F}$. It is shown that $W^\sigma(t)$ satisfies the following system of differential equations with mean derivatives:

$$D_* D_* W^\sigma(t) = \overline{F}, \qquad D_* W^\sigma(t) = 2D_{\mathrm{S}} W^\sigma(t).$$

We let $D_* W^{(\sigma)}(t) = u(t)_{W^{(\sigma)}(t)}$ translate, by right shifts, all $u(t)_{W^{(\sigma)}(t)}$ to the tangent space in the unit e of the group. After this translation, the conditional expectation involved in the definition of the mean derivative is carried over into the ordinary expectation. Thus, in the tangent space in unit, we obtain a deterministic curve $u_e(t)$, which is a nonautonomous vector field on the torus.

THEOREM. *The curve $u_e(t)$ on the torus satisfies the Burgers equation with viscosity coefficient $\sigma^2/2$ and external force F, and also obeys the continuity equation.*

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A. A. Gushchin (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia), **M. A. Nedoshivin** (Lomonosov Moscow State University, Moscow, Russia). **On Perkins' construction in the Skorokhod embedding problem.** [9]

Let Π denote the class of joint laws $\mathrm{Law}(B_\tau, \overline{B}_\tau)$ for the value of a Brownian motion $B = (B_t)_{t \geqslant 0}$ at a finite stopping time τ and its maximum $\overline{B}_\tau := \sup_{t \leqslant \tau} B_t$ in the time interval from 0 to τ. For an explicit description of this class, see [1] and [2]. Given an arbitrary measure μ on $\mathbf{R}$, we pose the problem of finding τ that minimizes the maximum $\overline{B}_\tau$, in the sense of stochastic ordering, among all τ with $\mathrm{Law}(B_\tau) = \mu$. The solution was found in [3] for μ with zero mean and in [4] in the general case. We supplement these results based only on the description of the set Π. Given a measure π, we denote by π_1 and π_2 its projections onto the coordinates.

THEOREM. *For any measure μ, the minimum $\nu := \min_{\pi \in \Pi:\, \pi_1 = \mu} \pi_2$ (in the sense of stochastic ordering) exists. In the class Π, there is only one distribution π with $\pi_1 = \mu$ and $\pi_2 = \nu$.*

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M. I. Ilolov, D. Sh. Rakhmatov (Tajik Academy of Sciences, Dushanbe, Tajikistan)**. The Cauchy problem for abstract fractional stochastic differential equations.**

In the Hilbert spaces H, H^1, consider the fractional Cauchy problem

$$D_t^\alpha X(t) = \big(AX(t) + F(t, X(t))\big)\,dt + B(t, X(t))\,dW(t), \quad t \in [0,T], \quad X(0) = \xi, \tag{1}$$

where $0 < \alpha < 1$, A is a nearly sectorial operator, the mappings $F(t,X)\colon [0,T]\times H \to H$, $B(t,X)\colon [0,T]\times H \to L_{\mathrm{HS}}(H, Q^{1/2}H^1)$ satisfy the Lipschitz condition and the linear growth condition, $W(t)$ is a Wiener process with values in $Q^{T/2}H^1$, and Q is a nonnegative trace operator in H^1. We are interested in the solution of problem (1) satisfying the condition

$$\mathbf{P}\biggl(\int_0^t \|X(s)\|^2\,ds < \infty\biggr) = 1. \tag{2}$$

The following theorem extends the results of [1].

THEOREM. *Let ξ be a measurable H-valued r.v. Then problem* (1) *has a solution, which is unique up to an equivalence among the processes satisfying* (2).

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A. V. Kolnogorov (Yaroslav-the-Wise Novgorod State University, Veliky Novgorod, Russia)**. Poissonian two-armed bandit: A Bayesian approach.** [10]

We consider a two-armed bandit problem on the control horizon $[0,T]$ with the incomes that follow the rule $e^{-\rho t}$, where t is the current time ($0 < T \leqslant +\infty$, $\rho > 0$). The intensities λ_1, λ_2 are given by the prior distribution density $\mu(\lambda_1,\lambda_2)$ on the set Θ. The Bayesian risk is equal to the minimum of the expectation of losses of the cumulative income with respect to the value attainable for known λ_1, λ_2.

THEOREM. *The Bayesian risk can be found by solving the partial differential equation*

$$\begin{aligned}\min\bigl(&R'_{t_1} + R(X_1+1, t_1, X_2, t_2) + e^{-\rho t} g^{(1)}(X_1,t_1,X_2,t_2),\\ &R'_{t_2} + R(X_1, t_1, X_2+1, t_2) + e^{-\rho t} g^{(2)}(X_1,t_1,X_2,t_2)\bigr) = 0\end{aligned}$$

backwards with the initial condition $R(X_1,t_1,X_2,t_2) = 0$ for $t_1 + t_2 = T$. The Bayesian strategy prescribes choosing, at time $t = t_1 + t_2$, the ℓth action if the ℓth term on the left-hand side of the equation is smaller. Here, t_1, t_2 are the current cumulative times of both actions' applications, and X_1, X_2 are the corresponding cumulative incomes,

$$g^{(1)}(X_1,t_1,X_2,t_2) = \iint_\Theta (\lambda_2 - \lambda_1)^+ \lambda_1^{X_1} e^{-\lambda_1 t_1} \lambda_2^{X_2} e^{-\lambda_2 t_2} \mu(\lambda_1,\lambda_2)\,d\lambda_1\,d\lambda_2,$$
$$g^{(2)}(X_1,t_1,X_2,t_2) = \iint_\Theta (\lambda_1 - \lambda_2)^+ \lambda_1^{X_1} e^{-\lambda_1 t_1} \lambda_2^{X_2} e^{-\lambda_2 t_2} \mu(\lambda_1,\lambda_2)\,d\lambda_1\,d\lambda_2.$$

The Bayesian risk is $R(0,0,0,0)$.

[10] Supported by the Russian Foundation for Basic Research (grant 20-01-00062).

For the results in the case $T < +\infty$, $\rho = 0$, see [1].

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E. V. Kudryavtsev, M. A. Fedotkin (National Research Lobachevsky State University of Nizhnii Novgorod, Russia). **Study of limit properties of a system of adaptive management of conflicting Cox–Lewis flows.**

We consider a queuing system described by a vector Markov sequence $\{(\Gamma_i, \kappa_{1,i}, \kappa_{2,i}); i \geqslant 0\}$ [1], [2]. Conflicting input flows of inhomogeneous customers were studied in [3]. Let $W_i(z_1, z_2)$, $i \geqslant 0$, be generating functions of one-dimensional distributions of the sequence $\{(\Gamma_i, \kappa_{1,i}, \kappa_{2,i});\ i \geqslant 0\}$. The following theorem holds.

Theorem. *If, for some $z_1, z_2 > 1$, the initial distribution of $\{(\Gamma_i, \kappa_{1,i}, \kappa_{2,i});\ i \geqslant 0\}$ satisfies $W_0(z_1, z_2) < \infty$, then, for existence of a limit distribution of this sequence, it is necessary and sufficient that the generating functions $W_{6i}(z_1, z_2)$ be bounded uniformly in i in some neighborhood of $(1,1)$.*

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O. E. Kudryavtsev (Russian Customs Academy, Rostov branch, Rostov-on-Don, Russia; Southern Federal University, Rostov-on-Don, Russia). **Pricing double barrier options under Lévy processes of unbounded variation.**

A method of simplified Wiener–Hopf factorization for pricing double barrier options

$$V(T,x) = \mathbf{E}^x \left[e^{-rT} \mathbf{1}_{\{\underline{X}_T > 0\}} \mathbf{1}_{\{\overline{X}_T < h\}} G(X_T) \right]$$

is constructed, where $G(x)$ is the payoff function, T is the maturity, h is the upper barrier, X_t is a Lévy process, and $\underline{X}_t = \inf_{0 \leqslant s \leqslant t} X_s$ and $\overline{X}_t = \sup_{0 \leqslant s \leqslant t} X_s$ are the processes of its infimum and supremum, respectively. The following theorem extends the results of [1] to the case of unbounded variation of jumps.

Theorem. *Let N be a sufficiently large natural number. We set $q = T/N$, $v_0(q,x) = G(x)\mathbf{1}_{(0,h)}(x)$ and define, for $n = 1, 2, \dots,$*

$$v_n(q,x) = \mathbf{E}^x \left[\frac{v_{n-1}(q, X_{T_{q+r}})}{1 + r/q} \mathbf{1}_{\{\underline{X}_{T_{q+r}} > 0\}} \mathbf{1}_{\{\overline{X}_{T_{q+r}} < h\}} \right],$$

where the random T_{q+r} has exponential distribution $\mathrm{Exp}(q+r)$*. Then, for a fixed x, the sequence $v_N(N/T, x)$ converges to $V(T,x)$ as $N \to \infty$.*

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V. A. Kutsenko, E. B. Yarovaya (Lomonosov Moscow State University, Moscow, Russia)**. Branching random walk in random environment with Gumbel potential.** [11]

We consider a branching random walk (BRW) on $\mathbf{Z}^d$ with continuous time in random environment. A BRW is based on a simple symmetric random walk. At each point $\mathbf{Z}^d$, a particle either dies or produces two offsprings with random intensities $b_0(\omega,x)$ and $b_2(\omega,x)$. In a fixed environment, the moments of the number of offsprings of a particle at a point x at time $t=0$ are random and denoted by $m_n(t,\omega,x)$ for the entire population and by $m_n(t,\omega,x,y)$ for the subpopulation at point y. We extend the proofs from [1] to BRWs with random environment (*potential*) $V(t,\omega,x)=b_2(\omega,x)-b_0(\omega,x)$ with Gumbel type distribution.

Theorem. *Let* $\ln\mathbf{P}(V>z)\sim -e^z$ *as* $z\to\infty$. *Then, for the moments* $\langle m_n^p\rangle$ *with initial conditions* $m_n(0,\cdot,y)=\delta_y(\cdot)$ *and* $m_n(0,\cdot)\equiv 1$,

$$\lim_{t\to\infty}\frac{\ln\langle m_n^p\rangle}{\ln\langle e^{pnVt}\rangle}=1,$$

where $n,p\in\mathbf{N}$, *and the angular brackets denote the expectation with respect to the probability measure generated by the random environment.*

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D. F. Kuznetsov (Peter the Great Saint-Petersburg Polytechnic University, Russia)**. A new approach to series expansion of iterated Stratonovich stochastic integrals of arbitrary multiplicity with respect to components of a multidimensional Wiener process.**

The following theorem is proved in [1, sections 2.10–2.15].

Theorem. *Let* $\psi_1(\tau),\ldots,\psi_k(\tau)\in C^1[t,T]$, *and let* $\{\phi_j(x)\}_{j=0}^{\infty}$ $(\phi_0(x)=1/\sqrt{T-t}$, $\phi_j(\tau)\in C[t,T])$ *be a basis in* $L_2[t,T]$ *such that the conditions* 1–3 *of Theorem* 2.30 *in* [1] *are met. Then*

$$J^*[\psi^{(k)}]_{T,t}^{(i_1\ldots i_k)}=\int_t^T\psi_k(t_k)\cdots\int_t^{t_2}\psi_1(t_1)\circ d\mathbf{W}_{t_1}^{(i_1)}\cdots\circ d\mathbf{W}_{t_k}^{(i_k)}=\underset{p\to\infty}{\text{l.i.m.}}\,S_{T,t}^{(i_1\ldots i_k)p},$$

where $S_{T,t}^{(i_1\ldots i_k)p}=\sum_{j_1,\ldots,j_k=0}^{p}C_{j_k\ldots j_1}\zeta_{j_1}^{(i_1)}\cdots\zeta_{j_k}^{(i_k)}$, $\zeta_j^{(i)}=\int_t^T\phi_j(\tau)\,d\mathbf{W}_\tau^{(i)}$ *are i.i.d.* $N(0,1)$*-r.v.'s* $(i\neq 0)$, $k\in\mathbf{N}$, $C_{j_k\ldots j_1}$ *is the Fourier coefficient corresponding to the kernels* $K(t_1,\ldots,t_k)=\psi_1(t_1)\cdots\psi_k(t_k)\mathbf{1}_{\{t_1<\cdots<t_k\}}$ $(k\geqslant 2)$ *and* $K(t_1)=\psi_1(t_1)$,

[11] Supported by the Russian Foundation for Basic Research (grant 20-01-00487).

$t_1, \dots, t_k \in [t,T]$, $i_1, \dots, i_k = 0, 1, \dots, m$, and $d\mathbf{W}_\tau^{(i)}$ and $\circ\, d\mathbf{W}_\tau^{(i)}$ are the Itô and Stratonovich differentials, respectively, $\mathbf{W}_\tau^{(0)} = \tau$. Moreover, for $\psi_1(\tau), \dots, \psi_k(\tau) \in C^1[t,T]$, we have $\mathbf{E}\big(J^*[\psi^{(k)}]_{T,t}^{(i_1\dots i_k)} - S_{T,t}^{(i_1\dots i_k)p}\big)^2 \leqslant C/p^{1-\varepsilon}$ for the case of Legendre polynomials and the Fourier basis, where $\varepsilon = 0$ (the Fourier basis for $k = 1, \dots, 5$ or polynomial basis for $k = 1, 2, 3$) or $\varepsilon > 0$ is arbitrarily small (a polynomial basis for $k = 4, 5$), and $C < \infty$ is independent of p.

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V. L. Litvinov (Samara State Technical University, Samara, Russia). **Stochastic longitudinal oscillations of a viscoelastic rope with moving boundaries with due account of damping forces.**

The wide use of mechanical objects with moving boundaries in engineering calls for the development of methods of their calculation. In the case of longitudinal oscillation, the principal effect on damping comes from elastic imperfections of the material of the oscillated object [1]. The study of viscoelasticity involves the analysis of stochastic stability of stochastic viscoelastic systems, their reliability, etc. We consider stochastic linear longitudinal oscillations of a viscoelastic rope with moving boundaries with due account of the damping forces. The initial conditions and the external load are considered random. To obtain the characteristics of the r.v.'s of stochastic oscillations, one has to find statistical estimates for the solution of a system of random integro–differential equations. To this end, the relaxation kernel can be taken as an exponential function with a random component. With the help of a difference kernel, one can reduce the problem to a system of stochastic differential equations. The coefficients are evaluated via the statistical numerical Monte Carlo method (see [2]).

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V. N. Litvinov, N. N. Gracheva, N. B. Rudenko (Don State Technical University, Rostov-on-Don, Russia; Azov-Black Sea Engineering Institute of Don State Agrarian University, Zernograd, Russia). **Probabilistic estimates of solutions of grid equations in heterogeneous computing systems.**[12]

The purpose of our study is to give a definition of functional dependencies of the execution time for solution of systems of linear algebraic equations (SLAEs) by a modified alternating-triangular iterative method (ATIM) on the dimension of fragments of a uniform 3D grid. Our studies are carried out for the most time-consuming stages of solving grid equations by ATIM, which involve solution of SLAEs with lower- and

[12]Supported by the Russian Science Foundation (grant 21-71-20050).

upper-triangular matrices [1] on the K-60 supercomputing system at Keldysh Institute of Applied Mathematics. Calculation time for solution of an SLAE is estimated via a sample characteristic of order statistics. The following result is proved.

THEOREM. *Calculation time for the lower-triangular stage of the solution of an SLAE by ATIM in parallel mode is given by* $T_{\mathrm{matm}} = \sum_{s=1}^{N_s} \max(\mathbf{T}_s)$, *where* s *and* N_s *are, respectively, the step number and the number of steps of a parallel-pipeline computational process, and* $\mathbf{T}_s$ *is the vector containing the values of time spent for evaluation of fragments of the grid for all calculations at step* s.

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